New Results on Passivity-based Pose Synchronization

Masayuki Fujita (Tokyo Institute of Technology)

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International Workshop on Recent Developments in Robotics and Control, in recognition of Mark W. Spong's leadership and contributions to the field on the occasion of his 60th birthday

Happy 60th Birthday, Mark !

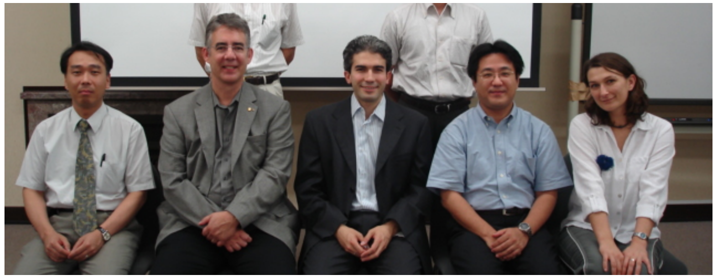
Mark Spong and MF at Orland, 2011CDC

Prologue: Output Synchronization

Tokyo Institute of Technology

In 2006, Mark visited us in TokyoTech and gave us a seminar on the O.S.:

N. Chopra and M.W. Spong [1]



Consider a Group of Passive Systems

$$\begin{cases} \dot{x}_i = f_i(x_i) + g_i(x_i)u_i \\ y_i = h_i(x_i) \end{cases}, \quad i \in \{1, \cdots, n\}$$

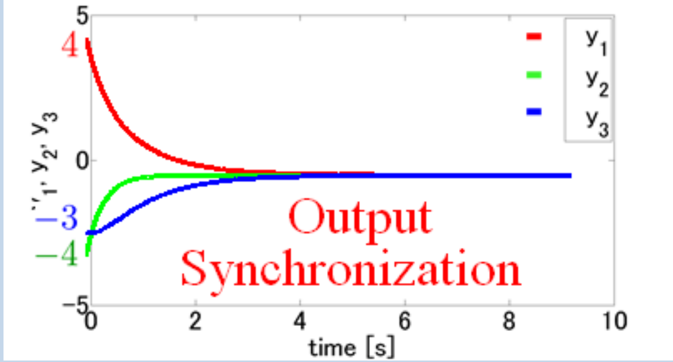
The system is assumed to be *passive*

$$^\exists V_i(x_i) \geq 0 \text{ s.t. } \dot{V}_i(x_i) \leq u_i^T y_i$$

Control Objective here is

Output Synchronization

$$\lim_{t \to \infty} \|y_i(t) - y_j(t)\| = 0$$

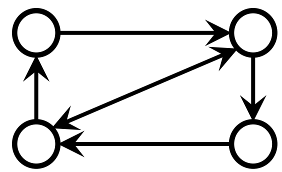


Synchronization law is given by the sum of relative output errors w.r.t. all neighbors

$$u_i = k \sum_{j \in \mathcal{N}_i} (y_j - y_i)$$

Relative Outputs

$\mathcal{N}_i$ neighbor agents



[1] N. Chopra and M. W. Spong, "Passivity-based Control of Multi-Agent Systems," *in Advance in Robot Control: From Everyday Physics to Human-Like Movements*, S. Kawamura and M. Svnin, eds., pp. 107—134, Springer, 2006.

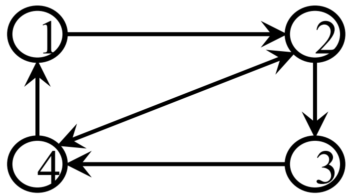
Prologue: Output Synchronization

By using this distributed law, output synchronization is proved.

Theorem[1] Under some assumptions, the present control law achieves output synchronization i.e.

$$\lim_{t\to\infty} \|y_i(t) - y_j(t)\| = 0 \ {}^{\forall} i, j \in \{1, \cdots, n\}$$

Technical Assumption: Interconnection topology among agents is fixed, balanced and *strongly connected*



An interesting feature in the proof is to use the sum of individual storage function (not relative) as a Lyapunov function candidate

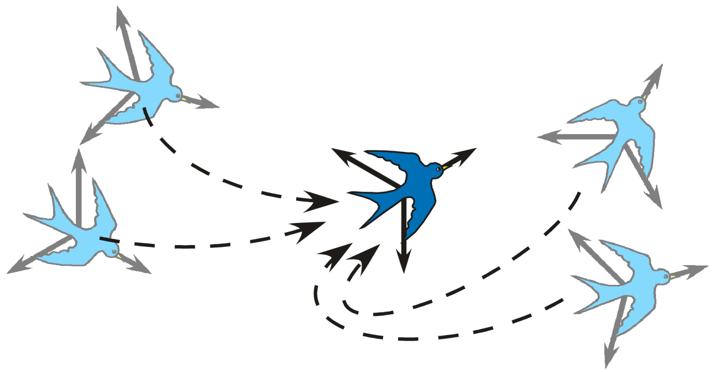
$$V = 2(V_1 + \cdots + V_n) \geq 0$$ (Sum of Individual Storage Functions)

$$\dot{V} = 2\sum_{i=1}^{n} u_i^T y_i = -k \sum_{i=1}^{n} \sum_{j\in\mathcal{N}_i} \|y_i - y_j\|^2 \leq 0$$ ⇒ LaSalle

From Passivity — From balanced digraphs

Outline

- Passivity-based Output Synchronization [1]
- Passivity-based Pose Synchronization on $SE(3)$ [2]
- Pose Synchronization with Vision



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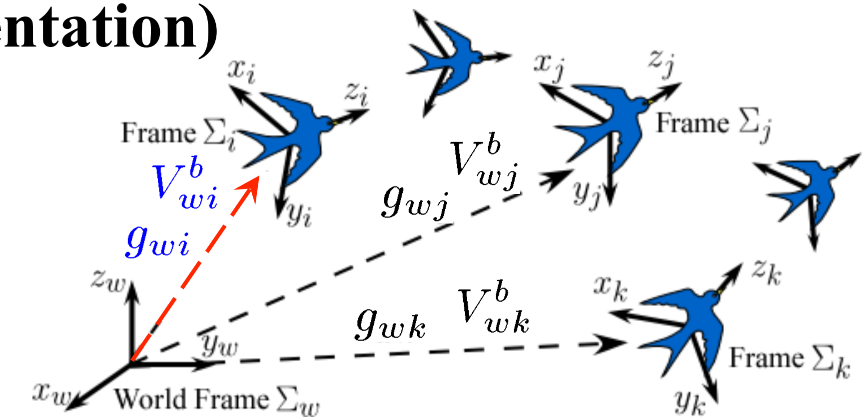
[2] T. Hatanaka, Y. Igarashi, M. Fujita and M. W. Spong, "Passivity-based Pose Synchronization in Three Dimensions," *IEEE Transactions on Automatic Control*, Vol. 57, No. 2, pp. 360-375, 2012.

Group of Rigid Bodies and Passivity

Let us consider 'pose' (positon & orientation) of rigid body *i* relative to Σ_w

$$(p_{wi}, e^{\hat{\xi}\theta_{wi}}) \in SE(3) \quad i = \{1, \cdots, n\}$$



Body velocity of rigid body *i*

$$V^b_{wi} := (g^{-1}_{wi}\dot{g}_{wi})^\vee = \begin{bmatrix} v^b_{wi} \\ \omega^b_{wi} \end{bmatrix} \in \mathcal{R}^6$$

Group of rigid bodies

$$\dot{g}_{wi} = g_{wi}\hat{V}^b_{wi} \quad (1)$$

and, throughout this talk, we view the velocity as control input to be determined. Then, pose evolution is described by (1)

Lemma: Rigid body motion (1) is passive in the sense of $\int_0^T (V^b_{wi}(t))^T \nu_i dt \geq -\beta, \ \beta > 0$

Storage Func.: $\Pi(g_{wi}) := \frac{1}{2}\|p_{wi}\|^2 + \phi(e^{\hat{\xi}\theta_{wi}}) \geq 0$

$$\phi(e^{\hat{\xi}\theta_{wi}}) := \frac{1}{2}\mathrm{tr}(I_3 - e^{\hat{\xi}\theta_{wi}})$$

Rigid Body Motion

V^b_{wi} → $\boxed{\dot{g}_{wi} = g_{wi}\hat{V}^b_{wi}}$ → ν_i

$$\nu_i := \begin{bmatrix} e^{-\hat{\xi}\theta_{wi}} p_{wi} \\ \mathrm{sk}(e^{\hat{\xi}\theta_{wi}})^\vee \end{bmatrix} \in \mathcal{R}^6$$

Pose Synchronization

We consider output synchronization on *SE*(3) based on the ref.[1]'s approach by letting the output be the pose of each body

A Group of Passive Systems

$$\begin{cases} \dot{x}_i = f_i(x_i) + g_i(x_i)u_i \\ y_i = h_i(x_i) \end{cases}$$

Control Objective
(Output Synchronization)

$$\lim_{t\to\infty} ||y_i(t) - y_j(t)|| = 0$$

A Group of Passive Systems

$$\dot{g}_{wi} = g_{wi}\hat{V}^b_{wi}$$

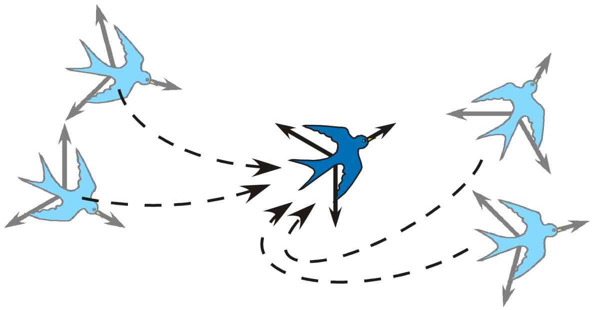
Output (Pose)

$$g_{wi} = \begin{bmatrix} e^{\hat{\xi}\theta_{wi}} & p_{wi} \\ 0 & 1 \end{bmatrix}$$

Control Objective (Pose Synchronization)

$$\lim_{t\to\infty} \Pi(g_{wi}^{-1}g_{wj}) = 0 \quad (2)$$

From the definition, pose synchronization means that both positions and orientations of all the rigid bodies converge to common values (or desired configurations)



Pose Synchronization

We next present a distributed control law based on [1]

Control Law [1]

$$u_i = k \sum_{j \in \mathcal{N}_i} (y_j - y_i)$$

Relative Output

Pose Synchronization Law (3)

$$V^b_{wi} = k_i \sum_{j \in \mathcal{N}_i} \left(\begin{bmatrix} e^{-\hat{\xi}\theta_{wi}}(p_{wj} - p_{wi}) \\ \mathrm{sk}(e^{-\hat{\xi}\theta_{wi}} e^{\hat{\xi}\theta_{wj}})^{\vee} \end{bmatrix} \right)$$

Relative Output

Then, we can prove the following theorem

Theorem 1 [2]: Pose Synchronization

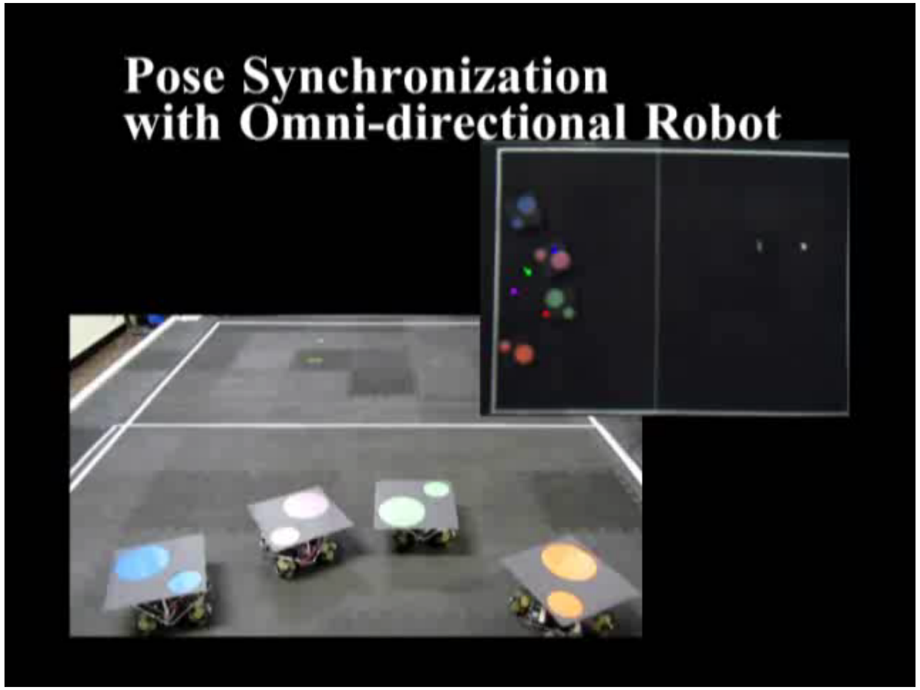
The present velocity input (3) achieves Pose Synchronization in the sense of (2) under some assumptions.

***Sketch of Proof*: As a potential function, we use the sum of individual storage functions, based on the passivity.**

$$U = \sum_{i=1}^{n} \frac{\gamma_i}{k_i} \left(\frac{1}{2} \|p_{wi}\|^2 + \phi(e^{\hat{\xi}\theta_{wi}}) \right) \Rightarrow \dot{U} \leq 0 \quad \text{(LaSalle)}$$

Experiments in Attitude/Pose Synch.

Attitude (only) Synchronization



Pose Synchronization

Flocking-like behavior successfully emerge by using the proposed control law

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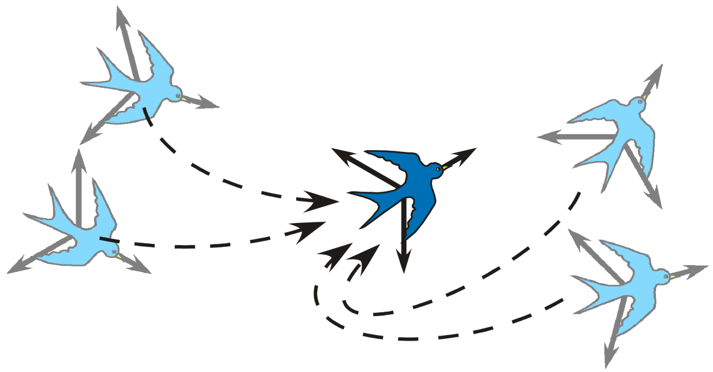

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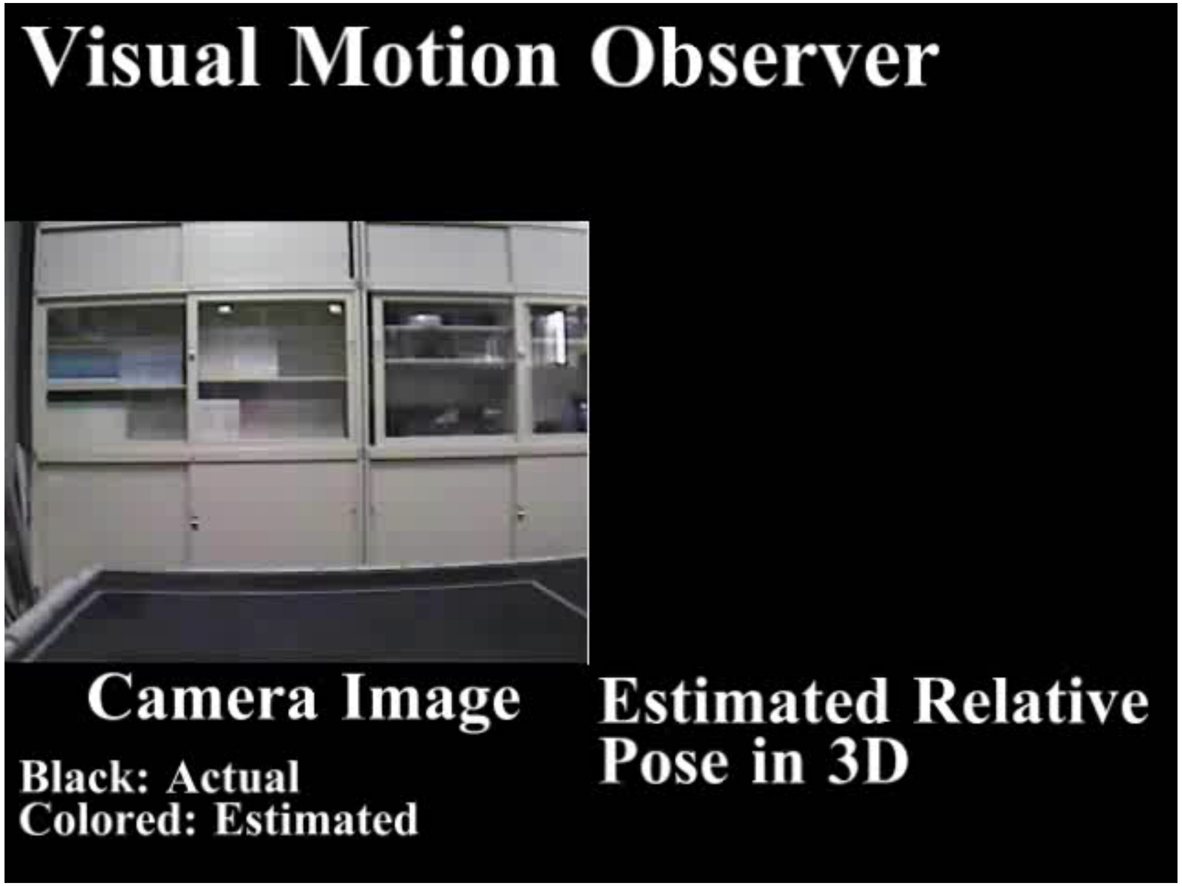
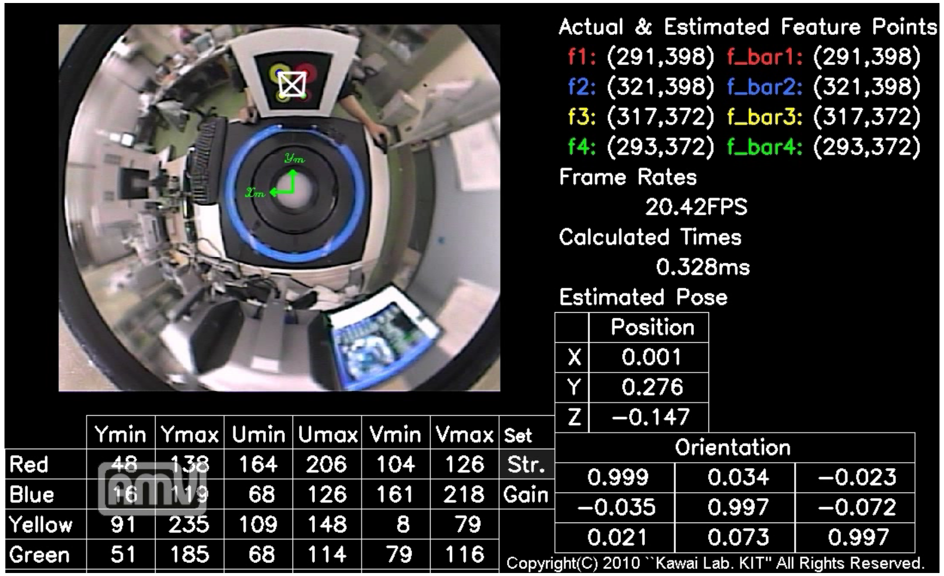
Passivity-based Visual Motion Observer [3]

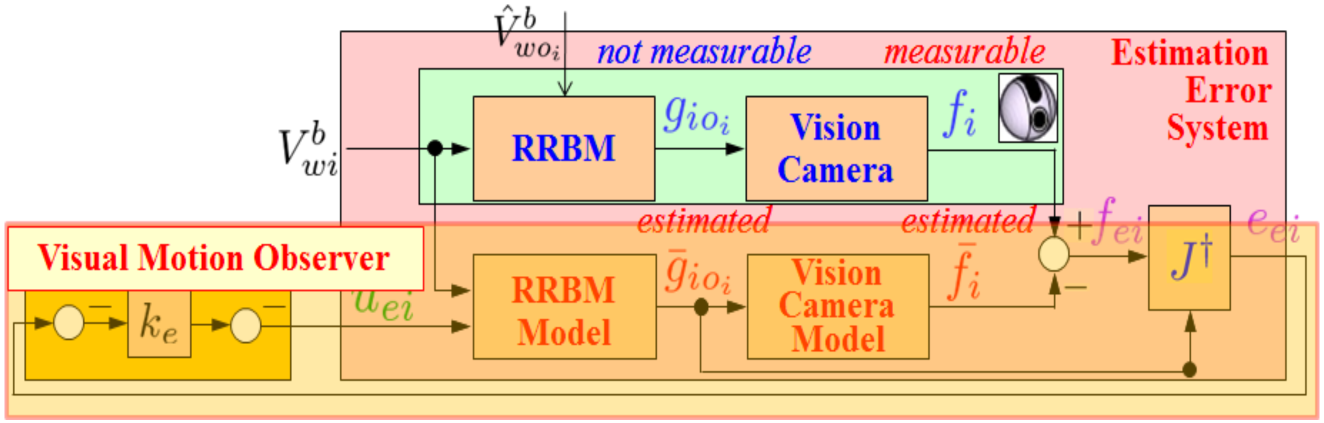


In 2007 [3], Mark and I proposed the Visual Motion Observer estimating a 3D target pose from visual measurement

[3] M. Fujita, H. Kawai and M.W. Spong, *IEEE TCST*, Vol, 15, No. 1, pp. 40-52, 2007.
(the 2008 IEEE TCST Outstanding Paper Award)

Passivity-based Visual Motion Observer [3]





cf. Luenberger

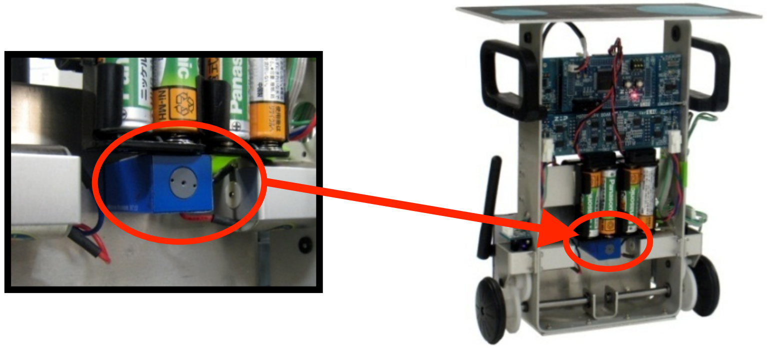
Passivity plays a key role

Vision-based Leader-Following

An example of Vision-based Leader-Following?

Centralized Sensing System

However



Fully distributed!
Control/Communication/Vision

(Passivity-based)
Attitude/Pose Synchronization

+

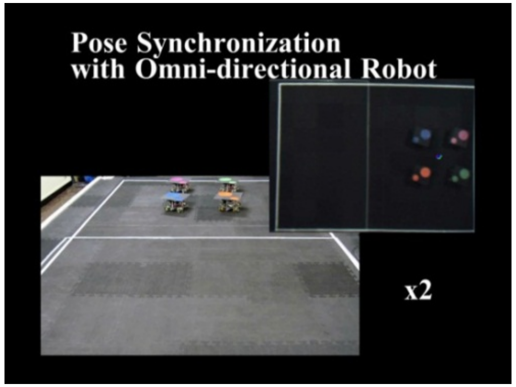
(Passivity-based)
Visual Motion Ovserver

Both of the works are based on passivity in rigid body motion.

[4] combines these two different works by the notion of Passivity.

[4] T. Ibuki, T. Hatanaka, M. Fujita and M.W. Spong, *IEEE the 50th CDC-ECC*, pp. 4999-5004, 2011.

Relative Information-based Synchronization



A new velocity input based only on *relative information* w.r.t. neighbors to produce natural behavior of the group

Pose Synchronization Law in [1]

$$V_{wi}^b = k_i \sum_{j \in \mathcal{N}_i} \left(\begin{bmatrix} e^{-\hat{\xi}\theta_{wi}}(p_{wj} - p_{wi}) \\ \mathrm{sk}(e^{-\hat{\xi}\theta_{wi}} e^{\hat{\xi}\theta_{wj}})^\vee \end{bmatrix} \right) + \begin{bmatrix} e^{-\hat{\xi}\theta_{wi}} v_d \\ e^{-\hat{\xi}\theta_{wi}} \omega_d \end{bmatrix}$$

Relative Information-based Law

$$V_{wi}^b = k_i \sum_{j \in \mathcal{N}_i} \left(\begin{bmatrix} e^{-\hat{\xi}\theta_{wi}}(p_{wj} - p_{wi}) \\ \mathrm{sk}(e^{-\hat{\xi}\theta_{wi}} e^{\hat{\xi}\theta_{wj}})^\vee \end{bmatrix} \right) + \begin{bmatrix} v_d \\ \omega_d \end{bmatrix} \quad (4)$$

common *Body* velocity

Theorem 2: Pose Synchronization[5]

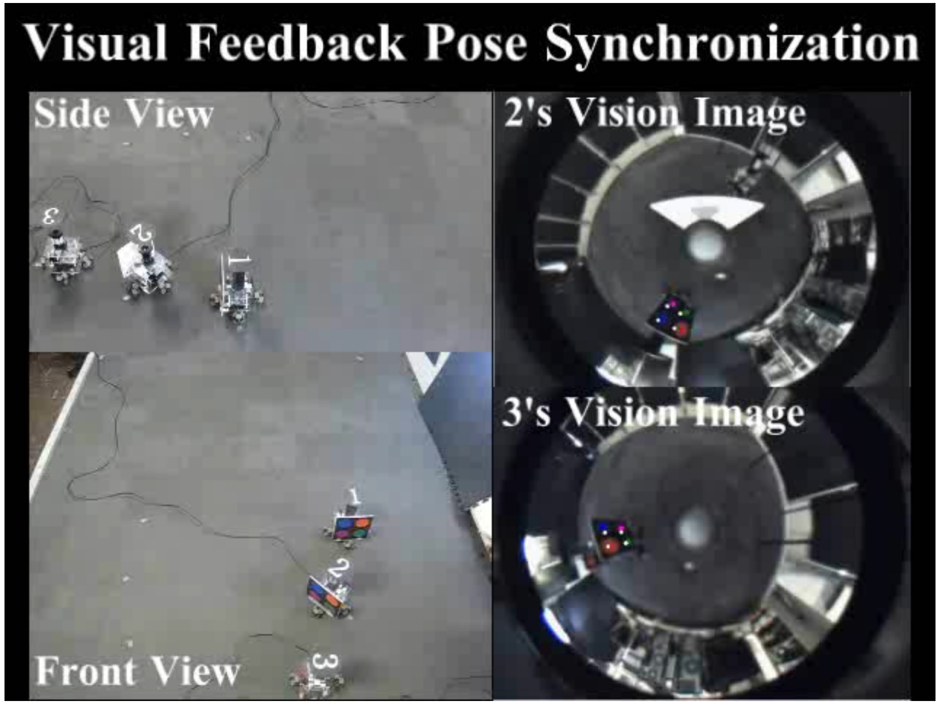
Velocity input (4) achieves Pose Sync. under some assumptions.

[5] T. Ibuki, T. Hatanaka and M. Fujita, Passivity-based Pose Synchronization Using Only Relative Pose Information under General Digraphs, Proc. of the 51st IEEE Conference on Decision and Control, (to be presented), 2012.

Passivity-based Visual Flocking

Visual Motion Observer　　　　Pose Synchronization

Combines these two different works by the notion of Passivity.



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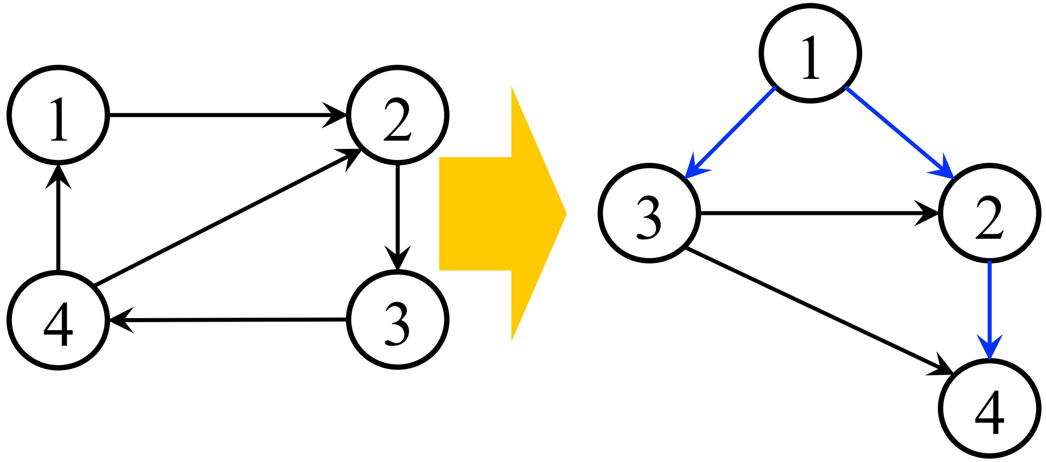
Extension to General Digraphs

A technique is available for proving synchronization for general *digraphs* containing spanning trees

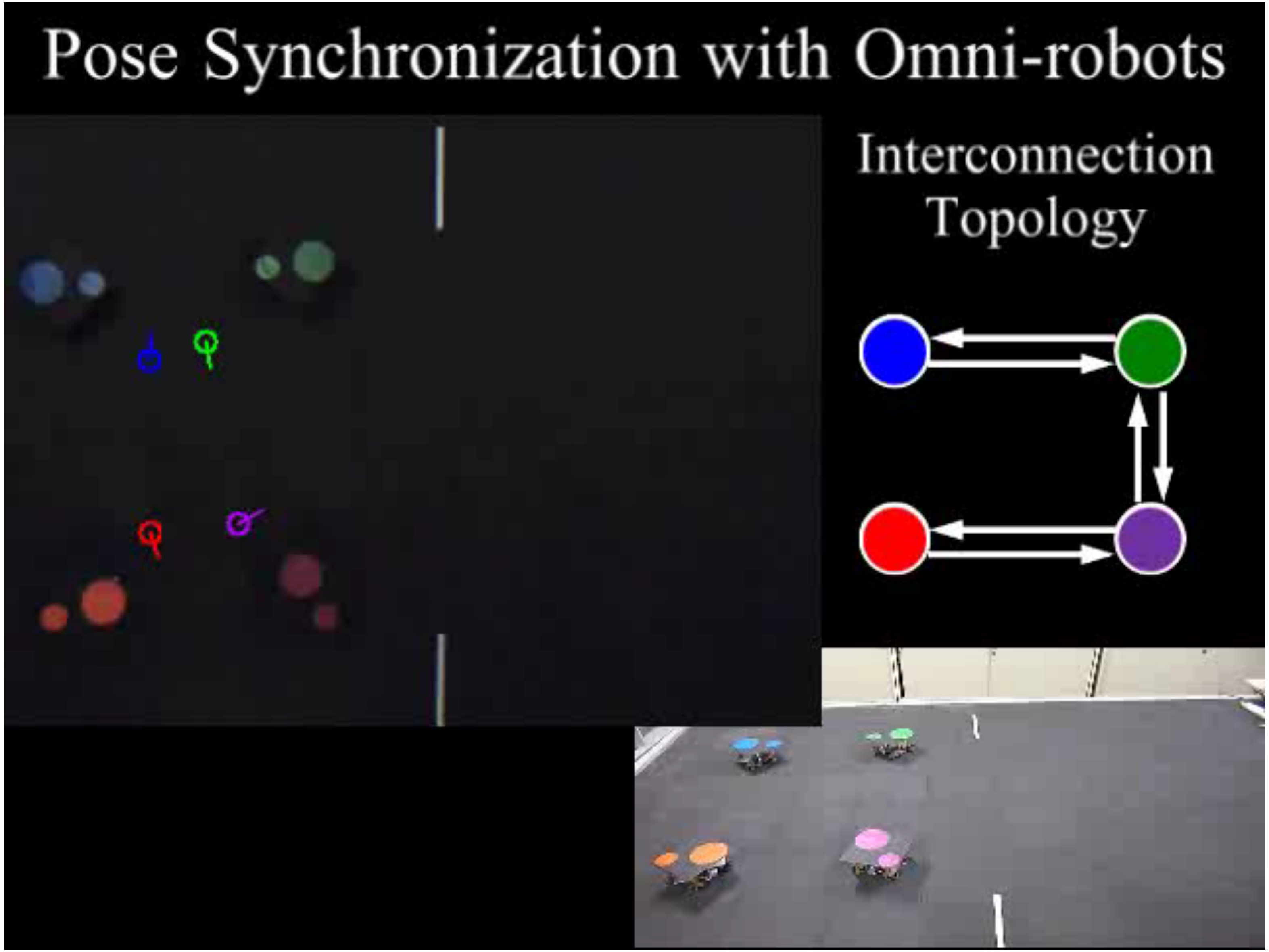


Theorem 3 [5]

The present velocity input (4) achieves Pose Synchronization for all initial states such that $e^{-\hat{\xi}\theta_{wi}}e^{\hat{\xi}\theta_{wj}} > 0 \ ^{\forall}i, j \in \mathcal{V}$ **if and only if fixed interconnection topologies contain a directed spanning tree**

[5] T. Ibuki, T. Hatanaka and M. Fujita, Passivity-based Pose Synchronization Using Only Relative Pose Information under General Digraphs, Proc. of the 51st IEEE Conference on Decision and Control, to be presented, 2012.

Experiment (General Digraphs)



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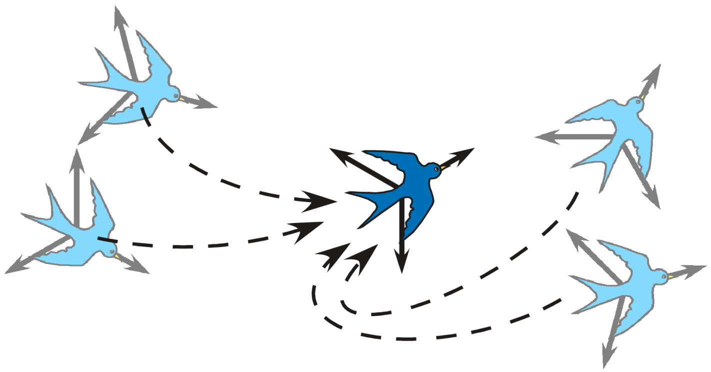

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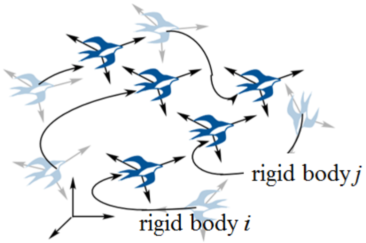
[2] T. Hatanaka, Y. Igarashi, M. Fujita and M. W. Spong, "Passivity-based Pose Synchronization in Three Dimensions," *IEEE Transactions on Automatic Control*, Vol. 57, No. 2, pp. 360-375, 2012.

Hands-on Experiments of 3D Synchronization

Birds Flocking in 2D



Extension to 3D Synchronization